Eur. Phys. J. A **25**, 159–164 (2005)
DOI 10.1140/epja/i2005-10089-y

THE EUROPEAN PHYSICAL JOURNAL A

The $K^-\alpha$ scattering length and the reaction $dd \to \alpha K^+K^-$

V.Yu. Grishina[1], L.A. Kondratyuk[2], A. Sibirtsev[3], M. Büscher[4], S. Krewald[4], U.-G. Meißner[3,4,a], and F.P. Sassen[3]

[1] Institute for Nuclear Research, 60th October Anniversary Prospect 7A, 117312 Moscow, Russia
[2] Institute of Theoretical and Experimental Physics, B. Cheremushkinskaya 25, 117259 Moscow, Russia
[3] Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Universität Bonn, Nußallee 14-16, D-53115 Bonn, Germany
[4] Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany

Received: 1 April 2005 / Revised version: 6 June 2005 /
Published online: 23 June 2005 –
Communicated by V. Vento

Abstract. We present predictions for the $K^-\alpha$ scattering length obtained within the framework of the multiple-scattering approach. Evaluating the pole position of the $K^-\alpha$ scattering amplitude within the zero-range approximation, we find a loosely bound $K^-\alpha$ state with a binding energy of $E_R = -2, \ldots, -7$ MeV and a width $\Gamma_R = 11, \ldots, 18$ MeV. We propose to measure the $K^-\alpha$ scattering length through the final-state interaction between the α and K^--meson produced in the reaction $dd \to \alpha K^+K^-$. It is found that the $K^-\alpha$ invariant-mass distribution from this reaction at energies near the threshold provides a new tool to determine the s-wave $K^-\alpha$ scattering length.

PACS. 25.10.+s Nuclear reactions involving few-nucleon systems – 13.75.-n Hadron-induced low- and intermediate-energy reactions and scattering (energy $\leq$ 10 GeV)

1 Introduction

Low-energy $\bar{K}N$ and $\bar{K}A$ interactions have gained substantial interest during the last two decades. It is known from the time-honored Martin analysis [1] that the isoscalar s-wave K^-N scattering length is large and repulsive, $\mathrm{Re}a_0 = -1.7$ fm, while the isovector length is moderately attractive, $\mathrm{Re}a_1 = 0.37$ fm. It is clear that such a strong repulsion in the $\bar{K}N$ isoscalar channel leads also to a repulsion in the low-energy K^-p system, since $\mathrm{Re}a_{K^-p} = 0.5\mathrm{Re}(a_0 + a_1) = -0.74$ fm. It should be noted that Conboy's analysis [2] of low-energy $\bar{K}N$ data gives a solution with $\mathrm{Re}a_0 = -1.03$ fm and $\mathrm{Re}a_1 = 0.94$ fm, that also results in repulsion in the K^-p channel, but with substantially smaller strength, $\mathrm{Re}a_{K^-p} = -0.05$ fm. Data from KEK show that the energy shift of the $1s$ level of kaonic hydrogen is repulsive [3,4]. Very recent results for kaonic hydrogen from the DEAR experiment [5] also indicate a repulsive energy shift. However, the consistency of the bound state with the scattering data can be questioned, as first pointed out in ref. [6].

Nevertheless, it is possible that the actual K^-p interaction is attractive if the isoscalar $\Lambda(1405)$-resonance is a bound state of the $\bar{K}N$ system [7,8]. A fundamental reason for such a scenario is provided by the leading-order term in the chiral expansion for the K^-N amplitude which is attractive. New developments in the analysis of the $\bar{K}N$ interaction based on chiral Lagrangians can be found in refs. [9–12]. These results provide further support for the description of the $\Lambda(1405)$ as a meson-baryon bound state. More recently, it has even been argued that there are indeed two poles in the complex plane in the vicinity of the nominal $\Lambda(1405)$ pole [13]. For recent evidence to support this scenario, see, *e.g.*, [14]. A different view seems to be taken in ref. [15].

Such a non-trivial dynamics of the $\bar{K}N$ interaction leads to very interesting in-medium phenomena in interactions of anti-kaons with finite nuclei as well as with dense nuclear matter, including neutron stars, see, *e.g.*, refs. [16–21].

Recently, exotic few-body nuclear systems involving the $\bar{K}$-meson as a constituent were studied by Akaishi and Yamazaki [22]. They proposed a phenomenological $\bar{K}N$ potential model, which reproduces the K^-p and K^-n scattering lengths from the Martin analysis [1], the kaonic hydrogen atom data from KEK [3,4] and the mass and width of the $\Lambda(1405)$-resonance. The $\bar{K}N$ interaction in this model is characterized by a strong $I = 0$ attraction, which allows the few-body systems to form dense nuclear objects. As a result, the nuclear ground states of a K^- in (pp), ^{3}He, ^{4}He and ^{8}Be, were predicted to be discrete states with binding energies of 48, 108, 86 and 113 MeV and widths of 61, 20, 34 and

[a] e-mail: meissner@itkp.uni-bonn.de

38 MeV, respectively. More recent work on this subject can be found, *e.g.*, in refs. [23,24].

Furthermore, very recently a strange tribaryon $S^0(3115)$ was detected in the interaction of stopped K^--mesons with ^{4}He [25]. Its width was found to be less than 21 MeV. In principle, this state may be interpreted as a candidate of a deeply bound state $(\bar{K}NNN)^{Z=0}$ with $I = 1$, $I_3 = -1$. However, the observed tribaryon $S^0(3115)$ is about 100 MeV lighter than the predicted mass. Moreover, in the experiment an isospin-1 state was detected at a position where no peak was predicted. It was discussed in ref. [26] that such discrepancy can be resolved by improvement of the model [22]. Nevertheless further searches for bound kaonic nuclear states as well as new data on the interactions of $\bar{K}$-mesons with lightest nuclei are thus of great importance.

Up to now the s-wave $K^-\alpha$ scattering length, which we denote as $A(K^-\alpha)$, has not been measured and relevant theoretical calculations have not yet been done. In this paper we present a first calculation of $A(K^-\alpha)$ within the framework of the multiple-scattering approach (MSA).

We investigate the pole position of the $K^-\alpha$ scattering amplitude within the zero-range approximation (ZRA) in order to find out whether the formation of a bound state in the $\bar{K}\alpha$ system is possible. Furthermore, we discuss the possibility to measure the $\bar{K}\alpha$ scattering length through the $\bar{K}\alpha$ final-state interaction (FSI). Recently it was proposed to measure the reaction $dd \to \alpha K^+K^-$ near the threshold at COSY-Jülich [27]. We apply our approach to calculate the $K^-\alpha$ FSI effect in this reaction and demonstrate that the $K^-\alpha$ invariant-mass distribution is sensitive enough to the $K^-\alpha$ FSI and may be used for the determination of the s-wave $K^-\alpha$ scattering length.

Our paper is organized as follows: In sect. 2 we calculate the $K^-\alpha$ scattering length within the MSA and determine the pole position of the amplitude in the zero-range approximation. In sect. 3 an analysis of the FSI in the reaction $dd \to \alpha K^+K^-$ is considered. Our conclusions are given in sect. 4.

2 The $K^-\alpha$ scattering length

2.1 Multiple-scattering formalism

To calculate the s-wave $K^-\alpha$ scattering length as well as the FSI enhancement factor, we use the Foldy-Brueckner adiabatic approach based on the multiple-scattering (MS) formalism [28]. Note that this method has already been used for the calculation of the enhancement factor in the reactions $pd \to {}^3\text{He}\eta$ [29], $pn \to d\eta$ [30] and $pp \to d\bar{K}^0K^+$ [31].

In the Foldy-Brueckner adiabatic approach, the continuum $K^-\alpha$ wave function, which is defined at fixed coordinates of the four nucleons in ^{4}He, can be written as the sum of the incident plane wave of the kaon and waves emerging from the four fixed scattering centers. Keeping only the s-wave contribution, we can express the total wave function Ψ_k through the j-channel wave functions $\psi_j(\mathbf{r}_j)$ in the following way:

$$\Psi_k(\mathbf{r}_{K^-};\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4) = \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}_{K^-}} + \sum_{j=1}^{4} t_{K^-N_j}\frac{\mathrm{e}^{ikR_j}}{R_j}\,\psi_j(\mathbf{r}_j), \quad (1)$$

where $R_j = |\mathbf{r}_{K^-} - \mathbf{r}_j|$ and the t-matrix, $t_{K^-N_j}$, is related to the elastic scattering amplitude f_{K^-N} via [30,31]

$$t_{K^-N}(k_{K^-N}) = \Big(1 + \frac{m_{K^-}}{m}\Big) f_{K^-N}(k_{K^-N}), \quad (2)$$

with m (m_{K^-}) the nucleon (charged-kaon) mass, and $k_{\bar{K}N}$ is the modulus of the relative $\bar{K}N$ momentum. Note that we use the unitarized scattering length approximation for the latter, *i.e.*

$$f^I_{\bar{K}N}(k_{\bar{K}N}) = \left[(a^I_{\bar{K}N})^{-1} - ik_{\bar{K}N}\right]^{-1}, \quad (3)$$

where I is the isospin of the $\bar{K}N$ system. For each scattering center j an effective wave $\psi_j(\mathbf{r}_j)$ is defined as the sum of the incident plane wave and the waves scattered from the three other centers,

$$\psi_j(\mathbf{r}_j) = \mathrm{e}^{i\mathbf{k}\cdot\mathbf{r}_j} + \sum_{l\neq j} t_{K^-N_l}\frac{\mathrm{e}^{ikR_{jl}}}{R_{jl}}\,\psi_l(\mathbf{r}_l)\ , \quad (4)$$

where $R_{jl} = |\mathbf{r}_l - \mathbf{r}_j|$. Therefore, the channel wave functions $\psi_j(\mathbf{r}_j)$ can be found by solving the system of the four linear equations (4).

To obtain the FSI factor we calculate the total wave function Ψ_k given by eq. (1) at $\mathbf{r}_{K^-} = \sum_{j=1}^4 \mathbf{r}_j = 0$ and average it over the coordinates of the nucleons $\mathbf{r}_j$ in ^{4}He. Thus, the FSI enhancement factor is [28]

$$\lambda^{\mathrm{MS}}(k_{K^-\alpha}) = \left|\left\langle \Psi_{q^{\mathrm{lab}}_{K^-}}\left(\mathbf{r}_{K^-} = \sum_{j=1}^{4}\mathbf{r}_j = 0;\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4\right)\right\rangle\right|^2. \quad (5)$$

For the nuclear density function we use the factorized form

$$|\Phi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)|^2 = \prod_{j=1}^{4}\rho_j(\mathbf{r}_j)\ , \quad (6)$$

where the single-nucleon density is taken in Gaussian form as

$$\rho(\mathbf{r}) = \frac{1}{(\pi R^2)^{3/2}}\ \mathrm{e}^{-r^2/R^2}\ , \quad (7)$$

with $R^2/4 = 0.62$ fm^2. Note that the independent particle model formulated by eqs. (6), (7) provides a rather good description of the ^{4}He electromagnetic form factor up to momentum transfer $\mathbf{q}^2 = 8$ fm^{-2} [32].

The integration in eq. (5) over the nucleon coordinates $\mathbf{r}_j$ was performed using the Monte Carlo method. This approach provides us with the possibility to include

Table 1. The $K^-\alpha$ scattering length for various sets of the elementary $\bar{K}N$ scattering lengths $a(\bar{K}N)$ $(I=0,1)$.

Set	Reference	$a_0(\bar{K}N)$ (fm)	$a_1(\bar{K}N)$ (fm)	$A(K^-\alpha)$ (fm)
1	[34]	$-1.59+i0.76$	$0.26+i.57$	$-1.80+i0.90$
2	[34]	$-1.61+i0.75$	$0.32+i0.70$	$-1.87+i0.95$
3	[7]	$-1.57+i0.78$	$0.32+i0.75$	$-1.90+i0.98$
4	[2]	$-1.03+i0.95$	$0.94+i0.72$	$-2.24+i1.58$
5	[11]	$-1.31+i1.24$	$0.26+i0.66$	$-1.98+i1.08$

all configurations of the nucleons in ^{4}He. Within this method we can also take into account in eq. (1) the dependence of the $t_{K^-N_j}$ amplitude on the type of nucleonic scatterer, *i.e.* proton or neutron. Note that the simple version of the multiple-scattering approach used in ref. [33] can be applied only to the case of identical scatterers.

The s-wave $K^-\alpha$ scattering length can be derived from the asymptotic expansion of eq. (1) at $r_{K^-}\to\infty$ and it is

$$A(K^-\alpha)=\frac{m_\alpha}{m_\alpha+m_{K^-}}\left\langle\sum_{j=1}^{4}t_{K^-N}\ \psi_j(\mathbf{r}_j)\right\rangle\Bigg|_{\sum_{j=1}^4\mathbf{r}_j=0}, \tag{8}$$

with m_α the α-particle mass. Here the procedure of averaging over the coordinates of the nucleons is similar to eq. (5).

2.2 S-wave scattering length and the pole position of the amplitude in the zero-range approximation

The basic uncertainties of the MSA calculations are given by the next-to-leading–order model corrections such as recoil corrections, contributions from inelastic double- and triple-scattering terms, *etc.* and due to the uncertainties of the elementary $I=0$ and $I=1$ $\bar{K}N$ scattering lengths. The calculations of the $K^-\alpha$ scattering length were done for five sets of parameters for the $\bar{K}N$ lengths shown in the table 1. Here we used the results from a K-matrix fit (Set 1) and a separable fit (Set 2) [34]. We also study the constant scattering length fit (CSL) given by Dalitz and Deloff [7], which we denoted as Set 3 and the CSL fit from Conboy [2] (Set 4). The recent predictions for $\bar{K}N$ scattering lengths based on the chiral unitary approach of ref. [11] are denoted as Set 5.

The results of our calculations are listed in the last column of table 1. These results are very similar for the Sets 1–3 giving the real and imaginary parts of the scattering length $A(K^-\alpha)$ within the range $-1.8,\ldots,-1.9$ fm and $0.9,\ldots,0.98$ fm, respectively. The results for Set 4 are quite different: $\mathrm{Re}A(K^-\alpha)=-2.24$ fm and $\mathrm{Im}A(K^-\alpha)=1.58$ fm. Furthermore, our calculations with Set 5 are close to the results obtained with Sets 1–3.

Unitarizing the constant scattering length, we can reconstruct the $\bar{K}\alpha$ scattering amplitude within the zero-range approximation (ZRA) as

$$f_{\bar{K}\alpha}(k)=\left[A(\bar{K}\alpha)^{-1}-ik\right]^{-1}, \tag{9}$$

where $k=k_{\bar{K}\alpha}$ is the relative momentum of the $K^-\alpha$ system. The denominator of the amplitude of eq. (9) has a zero at the complex energy

$$E^*=E_R-\frac{1}{2}i\Gamma_R=\frac{k^2}{2\mu}, \tag{10}$$

where E_R and Γ_R are the binding energy and width of the possible $K^-\alpha$ resonance, respectively. Here μ is reduced mass of the system with α mass taken as 3.728 GeV.

For Set 1 and Set 4 we find a pole at the complex energies of $E^*=(-6.7-i18/2)$ MeV and $E^*=(-2.0-i11.3/2)$ MeV, respectively. The calculations with Set 5 also result in a loosely bound state, $E^*=(-4.8-i14.9/2)$ MeV. Note that assuming a strongly attractive phenomenological $\bar{K}N$ potential, Akaishi and Yamazaki [22] predicted a deeply bound $\bar{K}\alpha$ state at $E^*=(-86-i34/2)$ MeV, which is far from our solutions. This problem can be clarified assuming that the loosely and deeply bound states are different eigenvalues of the $\bar{K}\alpha$ effective Hamiltonian. Our model for the $\bar{K}\alpha$ scattering amplitude is valid only near the threshold, *i.e.* when $kA(\bar{K}\alpha)\ll 1$. The ZRA cannot be applied for the description of deeply bound states when the pole of the scattering amplitude is located far away from the threshold. If the same procedure were applied to the $K^-\,{}^3$H system, we would find a similar loosely bound state. This state together with recently discovered deeply bound state, the $S^0(3115)$, can be considered as different eigenvalues of the $K^-\,{}^3$H effective Hamiltonian. In any case it is very important to measure the s-wave $\bar{K}\alpha$ scattering length experimentally and to clarify the situation concerning the possible existence of bound $\bar{K}\alpha$ states.

3 The reaction dd $\to \alpha$K$^-$K$^+$ near threshold and the K$^-\alpha$ final-state interaction

It is well known [27,35] that the reaction

$$dd\to\alpha K^-K^+ \tag{11}$$

provides an opportunity to study $I=0$ mesonic resonances in the K^-K^+ sector.

At the same time, near the reaction threshold it might be sensitive to the $K^-\alpha$ final-state interaction. Here we study whether it is possible to evaluate the s-wave $K^-\alpha$ scattering length from the $K^-\alpha$ final-state interaction. A similar evaluation of the $d\bar{K}^0$ FSI and relevant scattering length was done in our previous study [36] of the

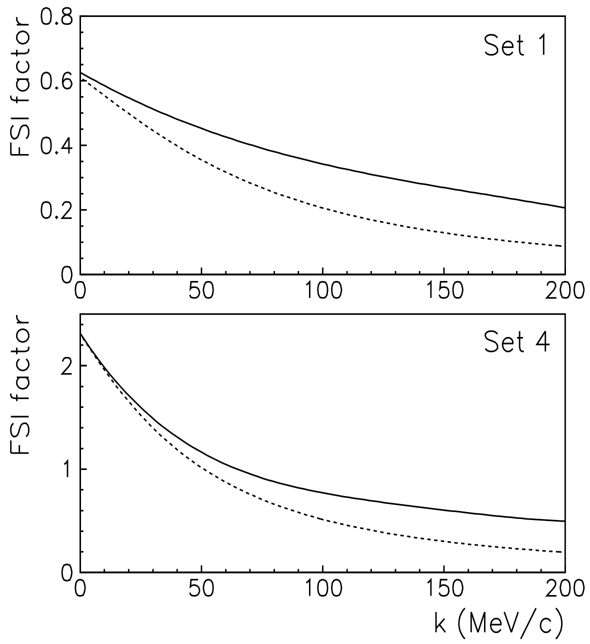

Fig. 1. The $K^-\alpha$ FSI enhancement factor $\lambda^{\rm MS}(k)$, eq. (5), as a function of the relative momentum k of the $K^-\alpha$ system. The solid lines in the lower and upper part of the figure show our calculations with Set 1 and Set 4 for the $\bar{K}N$ scattering lengths, respectively. The dashed lines illustrate the Watson-Migdal enhancement factor normalized to $\lambda^{\rm MS}(k)$ at $k = 0$.

$pp \to d\bar{K}^0K^+$ reaction. As has been stressed in ref. [37] this reaction should be very sensitive to the $\bar{K}^0d$ FSI. Through our analysis we extracted a new limit for the K^-d scattering length from the $\bar{K}^0d$ invariant-mass spectrum from the $pp \to d\bar{K}^0K^+$ reaction measured recently at COSY-Jülich [38].

It is clear that the FSI effect is essential at low invariant masses of the interacting particles, where the relative s-wave contribution is expected to be dominant. One can also safely assume that the range of the FSI is much larger as compared to the range of the basic hard interaction related to the production of the $\bar{K}K$-meson pair. This means that the basic production amplitude and the FSI term can be factorized [28,33,39–42] and the FSI can be taken into account by multiplying the production operator by the FSI enhancement factor defined by eq. (5).

Figure 1 shows the dependence of the $K^-\alpha$ FSI enhancement factor $\lambda^{\rm MS}(k)$ given by eq. (5) on the relative momentum of the $K^-\alpha$ system, k. The solid lines in the upper (lower) part of fig. 1 show the results obtained with Set 1 (Set 4) for the $\bar{K}N$ scattering length. The calculations with Set 1 result in $\lambda^{\rm MS}(k) \simeq 0.55$ at $k = 0$ and the FSI factor smoothly decreases with k. The calculations with Set 4 give $\lambda^{\rm MS}(k) > 1$ at $k = 0$ and show a much stronger k-dependence.

Following the Watson-Migdal approximation [43,44] the k-dependence of the enhancement factor is generally

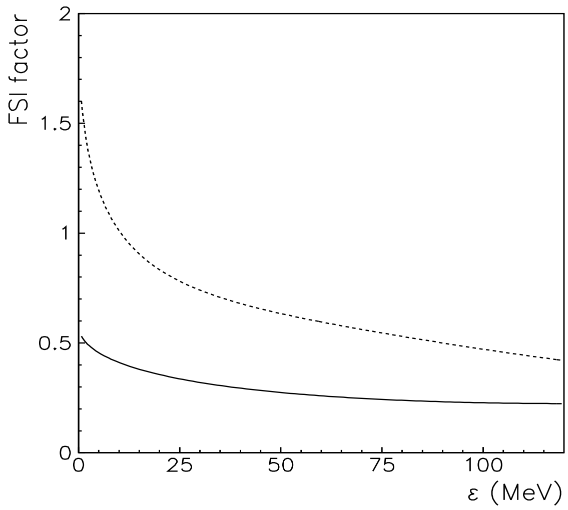

Fig. 2. The $K^-\alpha$ FSI factor averaged over the three-body phase space of the reaction $dd \to \alpha K^+K^-$ as a function of the excess energy. The solid and dashed lines show the calculations with parameters of Set 1 and 4, respectively.

described in terms of the on-shell scattering amplitude as

$$\lambda^{\rm WM} = \frac{C}{|1 - ikA_{\bar{K}\alpha}|^2}, \tag{12}$$

where C is the normalization constant.

Now, the dashed lines in fig. 1 illustrate the Watson-Migdal enhancement factor normalized to $\lambda^{\rm MS}(k)$ at $k=0$. The upper and lower parts of fig. 1 are calculated using the scattering lengths $A_{\bar{K}\alpha}$ obtained with parameters of Set 1 (Set 4), respectively, and listed in table 1. It is clear that the momentum dependence of $\lambda^{\rm WM}(k)$ and $\lambda^{\rm MS}(k)$ is different at different k. However, the absolute difference between $\lambda^{\rm WM}(k)$ and $\lambda^{\rm MS}(k)$ at $k \leq 100$ MeV/c is relatively small.

Obviously, the energy dependence of the total cross-section for the $dd \to \alpha K^+K^-$ reaction is also distorted by the $K^-\alpha$ FSI. In fig. 2 we show the enhancement factor $\lambda^{\rm MS}(k)$ averaged over the 3-body phase space as a function of the excess energy ϵ for the $dd \to \alpha K^+K^-$ reaction. The results for Sets 2, 3 and 5 are practically the same as for Set 1. It is interesting to note that there is essentially enhancement of the cross-section at small ϵ for Set 4, while for Set 1 we obtain suppression. The experiment would provide only a convolution of the production amplitude and FSI factor. Since the production amplitude is model dependent it is difficult to extract the absolute value of the FSI factor from the data. However, the dependence of the FSI on the relative momentum k is very well defined because the dependence of the basic hard interaction on k can be neglected at small k. According to ref. [27] the total cross-section of the reaction $dd \to \alpha K^+K^-$ might be about $0.4,\ldots,1$ nb at $\epsilon = 40,\ldots,50$ MeV.

Finally, we calculated the $K^-\alpha$ invariant-mass spectra at excess energies $\epsilon = 30$ and 50 MeV which are shown in fig. 3. The solid lines show the calculations for the pure phase space, *i.e.* for the constant production amplitude

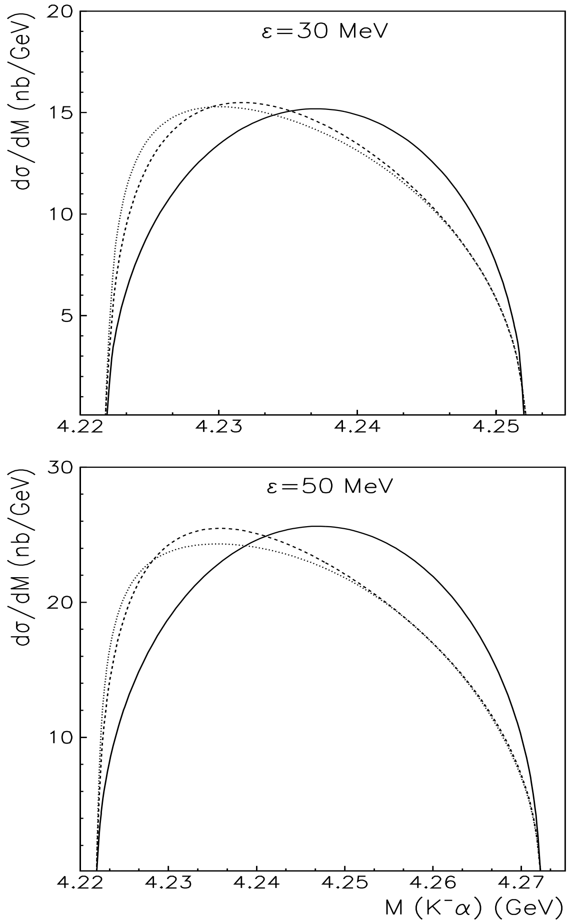

Fig. 3. The invariant $K^-\alpha$ mass spectra produced in the $dd \to \alpha K^+K^-$ reaction at excess energies 30 and 50 MeV. The solid lines describe the pure phase space distribution, while the dashed and dotted lines show our calculations with the $K^-\alpha$ FSI given by parameters of Set 1 and 4, respectively.

and neglecting FSI. The dashed and dotted lines in fig. 3 show the results obtained with the $K^-\alpha$ FSI calculated with the parameters of Set 1 and 4, respectively. All lines in each figure are normalized to the same value, given by the reaction cross-section at a certain excess energy. At $\epsilon = 50$ MeV the invariant-mass spectra are normalized to the $dd \to \alpha K^+K^-$ cross-section of 1 nb. It is clear that the FSI significantly changes the $K^-\alpha$ mass spectra. The most pronounced effect is observed at low invariant masses available in the first 10 MeV bin.

To draw quantitative conclusions, one can compare the ratio of the cross-sections at the lowest $K^-\alpha$ invariant masses, within the first 10 MeV bin, calculated with and without FSI. We found that this ratio $R = 1.26, \ldots, 1.34$ at $\epsilon = 30$ MeV, $1.49, \ldots, 1.56$ at $\epsilon = 50$ MeV and $1.84, \ldots, 2.18$ at $\epsilon = 100$ MeV. Here the limits of the ratio at each excess energy are given by the calculations with the $\bar{K}N$ scattering length from Set 1 and Set 4. With these estimates it is clear that the reasonable determination of the $K^-\alpha$ scattering length requires sufficient statistical accuracy at $K^-\alpha$ invariant masses below 4.23 GeV, at least 100 events. Such a high-precision experiment apparently can be done at COSY.

4 Conclusions

The findings of this study can be summarized as follows:

- We have investigated the s-wave $K^-\alpha$ scattering length and the $K^-\alpha$ FSI enhancement factor within the Foldy-Brueckner adiabatic approach based on the multiple-scattering formalism. We have studied uncertainties of the calculations due to the elementary K^-N scattering length presently available. The resulting s-wave $K^-\alpha$ scattering lengths for the various input parameters are collected in table 1.
- Through the determination of the pole position of the $K^-\alpha$ scattering amplitude within the zero-range approximation, we found a loosely bound state with binding energy $E_R = -2, \ldots, -7$ MeV and width $\Gamma_R = 11, \ldots, 18$ MeV. Our result for the loosely bound state can be considered as a different eigenvalue of the $K^-\alpha$ effective Hamiltonian as compared to the predictions of Akaishi and Yamazaki [22].
- We have analyzed the $K^-\alpha$ FSI in the reaction $dd \to \alpha K^+K^-$ and discussed the possibility to evaluate the $K^-\alpha$ scattering length from the $K^-\alpha$ invariant-mass spectra. We have demonstrated that the measurement of the $K^-\alpha$ mass distribution near the reaction threshold may provide a new tool for the determination of the s-wave $K^-\alpha$ scattering length.
- Furthermore, we have investigated the momentum dependence of the enhancement factor $\lambda^{\mathrm{MS}}(k)$ calculated within the MSA and compared it with the one obtained utilizing the Watson-Migdal formalism. It was found that the absolute difference between both calculations is relatively small at momenta $q \leq 100$ MeV/c.

It is important to stress that for kaonic helium atoms, energy shifts can be measured for the $2p$ state and widths for the $2p$ and $3d$ states. The $np \to 1s$ transitions for ^{4}He cannot be observed since the absorption from the p states is almost complete [45]. Therefore, the possibility to determine the s-wave $\bar{K}\alpha$ scattering length from experiments with kaonic atoms is questionable. With this respect a measurement at COSY provides a unique opportunity to determine the s-wave $K^-\alpha$ scattering length.

We appreciate discussions with C. Hanhart, M. Hartmann, R. Lemmer and P. Winter. We also thank A. Gal for a useful communication. This work was partially supported by the Deutsche Forschungsgemeinschaft through funds provided to the SFB/TR 16 "Subnuclear Structure of Matter" and by the DFG grant 436 RUS 113/787. This research is part of the EU Integrated Infrastructure Initiative Hadron Physics Project under contract No. RII3-CT-2004-506078. V.G. acknowledges support by the COSY FFE grant No. 41520739 and A.S. acknowledges support by the COSY FFE grant No. 41445400 (COSY-067).

References

1. A.D. Martin, Nucl. Phys. B **179**, 33 (1981).
2. J.E. Conboy, Rutherford-Appleton Lab. Report, RAL-85-091 (1985).
3. T.M. Itoh *et al.*, Phys. Rev. C **58**, 2366 (1998).
4. M. Iwasaki *et al.*, Phys. Rev. Lett. **78**, 3067 (1997).
5. C. Guaraldo *et al.*, Eur. Phys. J. A **19**, 185 (2004).
6. U.-G. Meißner, U. Raha, A. Rusetsky, Eur. Phys. J. C **35**, 349 (2004), hep-ph/0402261.
7. R.H. Dalitz, T.C. Wong, G. Rajasekaran, Phys. Rev. **153**, 1617 (1967).
8. P.B. Siegel, W. Weise, Phys. Rev. C **38**, 2221 (1988).
9. T. Waas, N. Kaiser, W. Weise, Phys. Lett. B **365**, 12 (1996); **379**, 34 (1996).
10. E. Oset, A. Ramos, Nucl. Phys. A **635**, 99 (1998), nucl-th/9711022.
11. J.A. Oller, U.-G. Meißner, Phys. Lett B **500**, 263 (2001), hep-ph/0011146.
12. M.F.M. Lutz, E.E. Kolomeitsev, Nucl. Phys. A **700**, 193 (2002), arXiv:nucl-th/0105042.
13. D. Jido, J.A. Oller, E. Oset, A. Ramos, U.-G. Meißner, Nucl. Phys. A **725**, 181 (2003), nucl-th/0303062.
14. V.K. Magas, E. Oset, A. Ramos, hep-ph/0503043.
15. B. Borasoy, R. Nißler, W. Weise, hep-ph/0410305.
16. A. Sibirtsev, W. Cassing, Nucl. Phys. A **641**, 476 (1998), nucl-th/9805021.
17. M. Lutz, Phys. Lett. B **426**, 12 (1998), nucl-th/9709073.
18. A. Sibirtsev, W. Cassing, Phys. Rev. C **61**, 057601 (2000).
19. A. Ramos *et al.*, Nucl. Phys. A **691**, 258c (2001), nucl-th/0101031.
20. H. Heiselberg, M. Hjorts-Jensen, Phys. Rep. **328**, 237 (2000), nucl-th/9902033.
21. A. Cieply, E. Friedman, A. Gal, J. Mares, Nucl. Phys. A **696**, 173 (2001), nucl-th/0104087.
22. Y. Akaishi, T. Yamazaki, Phys. Rev. C **65**, 044005 (2002).
23. A. Dote, H. Horiuchi, Y. Akaishi, T. Yamazaki, Phys. Rev. C **70** 044313 (2004), nucl-th/0309062.
24. A. Dote, Y. Akaishi, T. Yamazaki, Prog. Theor. Phys. Suppl. **156**, 184 (2004).
25. T. Suzuki *et al.*, Phys. Lett B **597**, 263 (2004).
26. Y. Akaishi, A. Dote, T. Yamazaki, Phys. Lett. B **613**, 140 (2005).
27. M. Büscher *et al.*, *Proceedings of the 1st COSY Summer School 2002*, hep-ph/0301126.
28. M.L. Goldberger, K.M. Watson, *Collision Theory* (John Wiley and Sons, Inc., New York, London, Sidney, 1964).
29. G. Fäldt, C. Wilkin, Nucl. Phys. A **587**, 769 (1995).
30. V.Yu. Grishina *et al.*, Phys. Lett. B **475**, 9 (2000), nucl-th/9905049.
31. V.Yu. Grishina, L.A. Kondratyuk, M. Büscher, W. Cassing, Eur. Phys. J. A **21**, 507 (2004), nucl-th/0402093.
32. V.N. Boitsov, L.A. Kondratyuk, V.B. Kopeliovich, Sov. J. Nucl. Phys. **16**, 287 (1973).
33. S. Wycech, A.M. Green, J.A. Niskanen, Phys. Rev. C **52**, 544 (1995).
34. R.C. Barrett, A. Deloff, Phys. Rev. C **60**, 025201 (1999).
35. V.Yu. Grishina, L.A. Kondratyuk, M. Büscher, W. Cassing, H. Ströher, Phys. Lett. B **521**, 217 (2001), nucl-th/0103081.
36. A.Sibirtsev *et al.*, Phys. Lett. B **601**, 132 (2004), nucl-th/0406061.
37. E. Oset, J.A. Oller, U.-G. Meißner, Eur. Phys. J. A **12**, 435 (2001), nucl-th/0109050.
38. V. Kleber *et al.*, Phys. Rev. Lett. **91**, 172304 (2003), nucl-ex/0304020.
39. A. Sibirtsev, Phys. Lett. B **359**, 29 (1995).
40. A. Sibirtsev, W. Cassing, Eur. Phys. J. A **2**, 333 (1998), nucl-th/9806022.
41. C. Hanhart, Phys. Rep. **397**, 155 (2004), hep-ph/0311341.
42. V.Yu. Grishina, L.A. Kondratyuk, E.L. Bratkovskaya, M. Büscher, W. Cassing, Eur. Phys. J. A **9**, 277 (2000), nucl-th/0007074.
43. K.M. Watson, Phys. Rev. **88**, 1163 (1952).
44. A.B. Migdal, Sov. Phys. JETP **1**, 2 (1955).
45. C.J. Batty, Nucl. Phys. A **508**, 89c (1990).